

Contents

Preface	ix
1 Introduction to Writing Proofs in Mathematics	1
1.1 Conditional Statements	1
1.2 Constructing Direct Proofs	13
1.3 Solutions for the Progress Checks	26
1.4 Chapter 1 Summary	27
2 Logical Reasoning	29
2.1 Statements and Logical Operators	29
2.2 Logically Equivalent Statements	37
2.3 Predicates, Sets, and Quantifiers	47
2.4 Quantifiers and Negations	58
2.5 Solutions for the Progress Checks	71
2.6 Chapter 2 Summary	74
3 Constructing and Writing Proofs in Mathematics	76
3.1 Direct Proofs	76
3.2 More Methods of Proof	93
3.3 Proof by Contradiction	107
3.4 Using Cases in Proofs	120
3.5 The Division Algorithm and Congruence	128
3.6 Solutions for the Progress Checks	142
3.7 Chapter 3 Summary	146
4 Set Theory	151
4.1 Operations on Sets	151
4.2 Proving Set Relationships	168
4.3 Properties of Set Operations	180
4.4 Cartesian Products	190

4.5	Indexed Families of Sets	199
4.6	Solutions for the Progress Checks	211
4.7	Chapter 4 Summary	215
5	Mathematical Induction	218
5.1	The Principle of Mathematical Induction	218
5.2	Other Forms of Mathematical Induction	235
5.3	Induction and Recursion	248
5.4	Solutions for the Progress Checks	259
5.5	Chapter 5 Summary	261
6	Functions	264
6.1	Introduction to Functions	264
6.2	More about Functions	278
6.3	Injections, Surjections, and Bijections	290
6.4	Composition of Functions	306
6.5	Inverse Functions	317
6.6	Functions Acting on Sets	333
6.7	Solutions for the Progress Checks	343
6.8	Chapter 6 Summary	346
7	Equivalence Relations	349
7.1	Relations	349
7.2	Equivalence Relations	359
7.3	Equivalence Classes	372
7.4	Modular Arithmetic	384
7.5	Solutions for the Progress Checks	395
7.6	Chapter 7 Summary	397
8	Topics in Number Theory	399
8.1	The Greatest Common Divisor	399
8.2	Prime Numbers and Prime Factorizations	410
8.3	Linear Diophantine Equations	422
8.4	Solutions for the Progress Checks	431
8.5	Chapter 8 Summary	435
9	Finite and Infinite Sets	437
9.1	Finite Sets	437
9.2	Countable Sets	446
9.3	Uncountable Sets	459

<i>Contents</i>	vii
9.4 Solutions for the Progress Checks	470
9.5 Chapter 9 Summary	473
A Guidelines for Writing Mathematical Proofs	475
B Answers and Hints for Selected Exercises	480
C List of Symbols	505
Index	508